Progress and Plans at USU

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PSI-Center Summer Meeting

Seattle, WA

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Recent papers.

- J.-Y. Ji and E. D. Held, “Electron parallel closures for arbitrary collisionality", *Phys. Plasmas*, **21**, 122115 (2014).
  - complete set of integral parallel electron closures with fitted kernels for $Z = 1$,
- J.-Y. Ji and E. D. Held, “Ion closure theory for high collisionality revisited", *Phys. Plasmas* **22**, 062114 (2015).
  - keeps ion-electron collision operator,
  - corrects collisional ion transport coefficients when $T_i > T_e$.
- E. D. Held, S. E. Kruger, J.-Y. Ji, E. A. Belli and B. C. Lyons, “Verification of continuum drift kinetic equation solvers in NIMROD", *Phys. Plasmas*, **22**, 032511 (2015).
  - NIMROD’s continuum electron and ion DKE solutions correctly predict neoclassical transport.

Integral (nonlocal) parallel closures [Ji and Held, PoP 21, 122116 (2014)]

$$n_{AB}(\eta) = \int d\eta' K_{AB}(\eta - \eta') g_B(\eta')$$ where $A, B = h, R, \pi$ and $d\eta = \frac{d\ell}{\lambda_{\mathrm{mfp}}}$

$$h_\parallel(\eta) = -\frac{1}{2} T v_T \int d\eta' K_{hh} \frac{n}{T} \frac{dT}{d\eta'} + T v_T \int d\eta' Z K_{hR} n \frac{V_{\mathrm{ei}\parallel}}{v_T} - T v_T \int d\eta' K_{h\pi} (\frac{3}{4} n \tau_{\mathrm{ee}} W_\parallel)$$

$$R_\parallel(\eta) = -\frac{mn}{\tau_{\mathrm{ei}}} V_{\mathrm{ei}\parallel} + \frac{m v_T}{\tau_{\mathrm{ei}}} \int d\eta' [-K_{Rh} \frac{n}{2T} \frac{dT}{d\eta'} + Z K_{RR} n \frac{V_{\mathrm{ei}\parallel}}{v_T} - K_{R\pi} (\frac{3}{4} n \tau_{\mathrm{ee}} W_\parallel)]$$

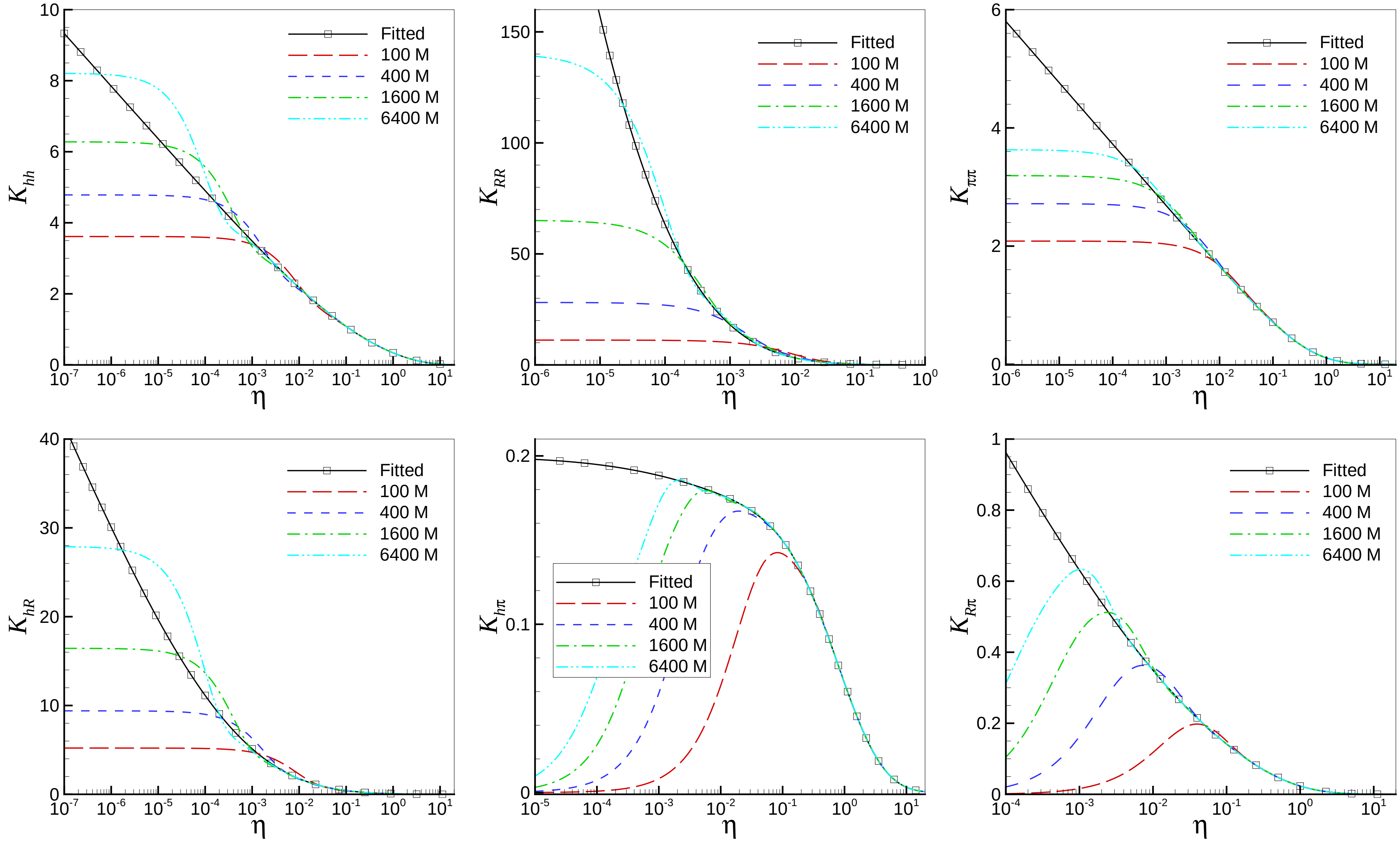
$$\pi_\parallel(\eta) = -T \int d\eta' K_{\pi h} \frac{n}{T} \frac{dT}{d\eta'} + 2T \int d\eta' Z K_{\pi R} n \frac{V_{\mathrm{ei}\parallel}}{v_T} - T \int d\eta' K_{\pi\pi} (\frac{3}{4} n \tau_{\mathrm{ee}} W_\parallel)$$

- Fitted kernel functions $(Z = 1)$

$$K_{AB}(\eta) = -[d + a \exp(-b\eta^c)] \ln[1 - \alpha \exp(-\beta \eta^\gamma)]$$

	a	b	c	d	α	β	γ
K_{hh}	-5.32	0.170	0.646	6.87	1	2.02	0.417
K_{hR}	6.37	5.12	0.160	0.100	1	1	0.583
$K_{h\pi}$	-0.229	2.26	0.594	0.363	0.775	1.49	0.478
K_{RR}	245	8.06	0.147	0.432	1	3.40	0.347
$K_{R\pi}$	-0.226	3.21	0.678	0.696	1	3.40	0.347
$K_{\pi\pi}$	0.724	0.932	0.654	0.195	1	1.60	0.491

Fitted kernel functions (6400 M + collisionless)



Extending to $Z = 2, \cdots, 10$ (K_{hh} only shown)

[In collaboration with S.-K. Kim and Y.-S Na]

- $K_{AB}(\eta) = -[d + a\exp(-b\eta^c)]\ln[1 - \alpha\exp(-\beta\eta^\gamma)]$

Z	1	2	3	4	5	6	7	8	9	10
a	-3.85	-3.61	-4.02	-4.50	-5.52	-6.98	-9.58	-14.8	-24.2	-39.0
b	0.248	.387	0.590	0.746	0.796	0.776	0.686	0.528	0.377	0.267
c	0.680	0.551	0.537	0.569	0.581	0.583	0.583	0.583	0.583	0.583
d	5.40	5.47	6.07	6.66	7.74	9.28	11.9	17.1	26.5	41.4
α	1	1	1	1	1	1	1	1	1	1
β	2.02	2.49	2.91	3.20	3.46	3.70	3.93	4.18	4.43	4.65
γ	0.417	0.348	0.316	0.300	0.289	0.281	0.279	0.277	0.276	0.275

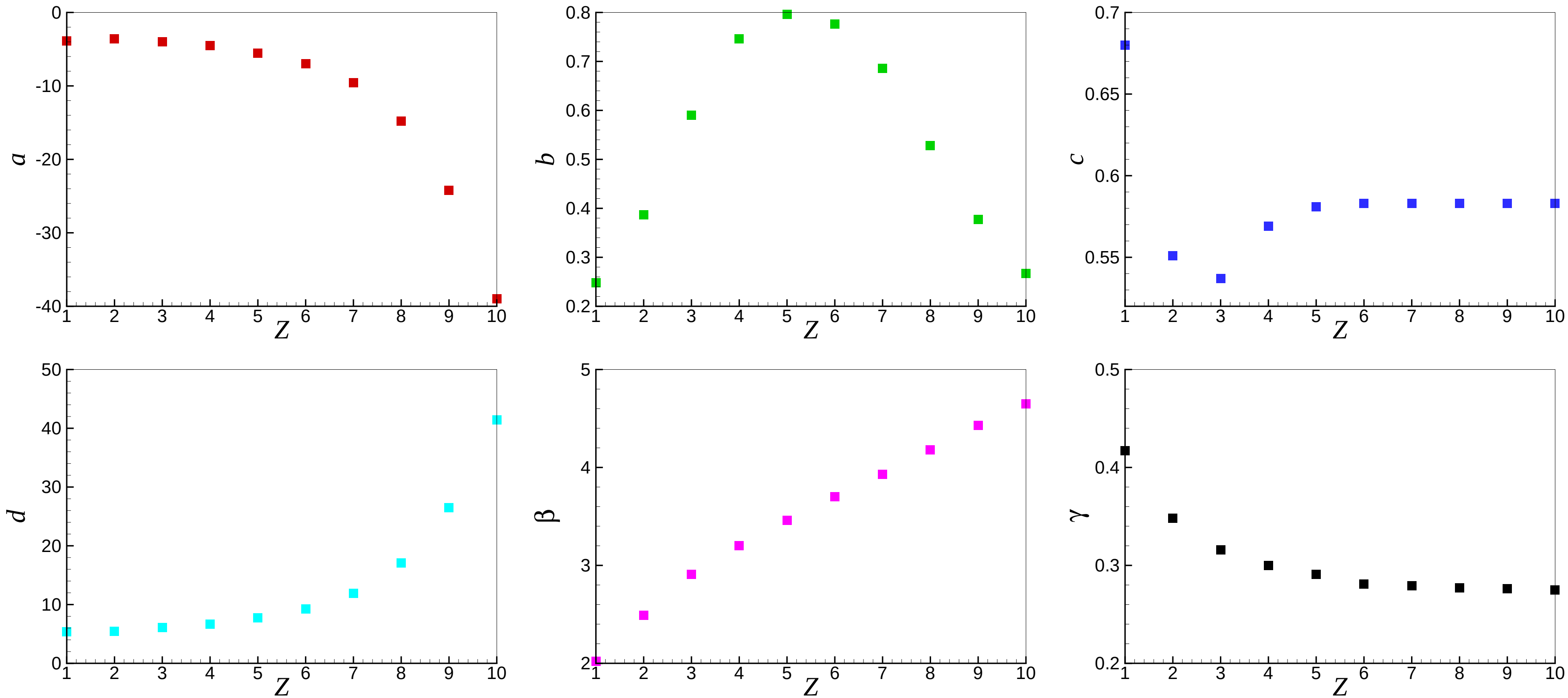
- Errors are less than 5 % in the convergent regime $(\lambda_{\mathrm{mfp}}/|\nabla^{-1}| \lesssim 80)$
- For $\eta \ll 1$, kernels approach the collisionless asymptote

$$K_{hh}(\eta) \approx -\frac{18}{5\pi^{3/2}}(\ln|\eta| + \text{const.})$$

$$a = \frac{18}{5\pi^{3/2}\gamma} - d$$

- $Z = 1$ parameters are slightly modified for better interpolation $1 < Z < 2$

Extending to $Z = 1, 2, \cdots, 10$ (K_{hh} only shown)



- Smooth change in Z
  $\Rightarrow$ Linear interpolations yield accurate closures for noninteger Z_{eff}

$$A_{Z\text{eff}} = (1+Z-Z_{\text{eff}})A_Z + (Z_{\text{eff}}-Z)A_{Z+1} \text{ for } A = b, c, d, \beta, \gamma \text{ and } Z \le Z_{\text{eff}} \le Z+1$$

Z	1	1.2	1.4	1.6	1.8	2	2.5
max. error	1.0%	1.5%	2.2%	2.6%	2.8%	2.8%	3.0%

Continuum solution of electron, ion and hot particle DKEs in NIMROD.

Continuum approach solves the drift kinetic equation (DKE):

$$\frac{\partial f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f - \frac{1-\xi^2}{2\xi}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln B \frac{\partial f}{\partial \xi} - \frac{s}{2}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln T_0 \frac{\partial f}{\partial s} - C(f) +$$

$$\frac{1-\xi^2}{2\xi}\left[\xi^2 \frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \frac{e}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + \xi^2 \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B\right] \frac{\partial f}{\partial \xi} +$$

$$\frac{s}{2}\left[-(1-\xi^2)\frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \frac{e}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + (1+\xi^2)\frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B\right] \frac{\partial f}{\partial s} = 0,$$

where the drift velocity

$$\mathbf{v}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{T_0 s^2}{qB^2}\left[\left(1+\xi^2\right) \mathbf{b} \times \nabla B + 2\xi^2 \mu_0 \mathbf{J}_\perp + (1-\xi^2)\mu_0 \mathbf{J}_{||}\right] + \frac{m v_0 s \xi}{qB^2} \mathbf{b} \times \frac{\partial \mathbf{B}}{\partial t}.$$

Use NIMROD's spatial representation and efficient 2D velocity space representation.

Distribution functions expanded as

$$f(R, Z, \phi, \xi, s, t) = \sum_i f_{i,n=0}(\xi, s, t)\alpha_{i,n=0} + \sum_{i,n>0} f_{i,n}(\xi, s, t)\alpha_{i,n} + f^*_{i,n}(\xi, s, t)\alpha^*_{i,n},$$

with 2D velocity space represented as

$$f_{i,n}(\xi, s, t) = \sum_l \sum_{k=0}^{N_s-1} f_{i,n,l,k}(t) P_l(\xi)\delta(s - s_k).$$

$P_l(\xi)$ are 1D FE in a pitch-angle type variable, ξ, and DKEs are solved at N_s collocation points in normalized speed, s.

Linearized DKE for hot particles is relatively simple.

Assuming $\mathbf{E}_{eq} = 0$:

$$\frac{\partial \delta f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D)_{eq} \cdot \nabla \delta f - \frac{1-\xi^2}{2\xi} \left((\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln B \right)_{eq} \frac{\partial \delta f}{\partial \xi} - \frac{s}{2} (\mathbf{v}_{||} + \mathbf{v}_D)_{eq} \cdot \nabla \ln T_0 \frac{\partial \delta f}{\partial s} =$$

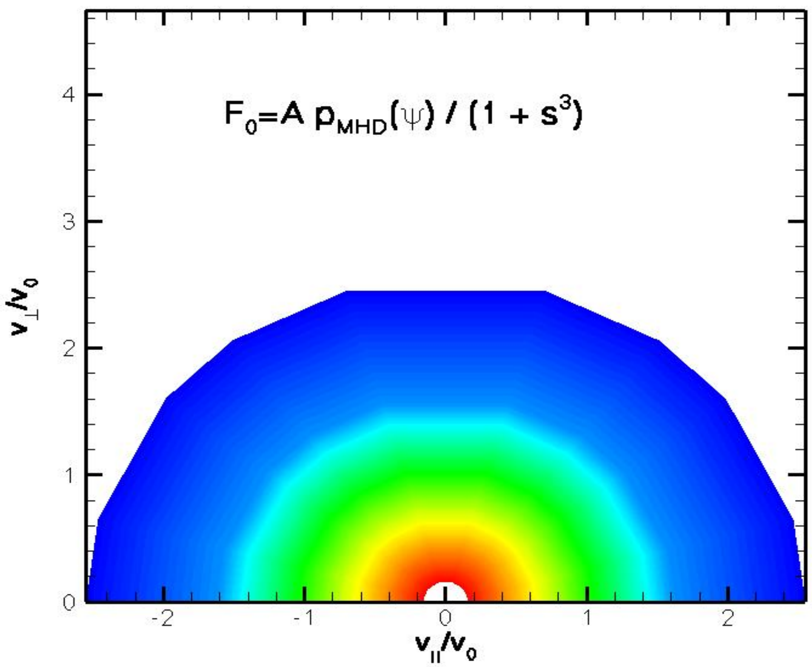
$$\delta(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f_0 - \frac{1-\xi^2}{2\xi} \delta \left((\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln B \right) \frac{\partial f_0}{\partial \xi} - \frac{s}{2} \delta(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln T_0 \frac{\partial f_0}{\partial s}$$

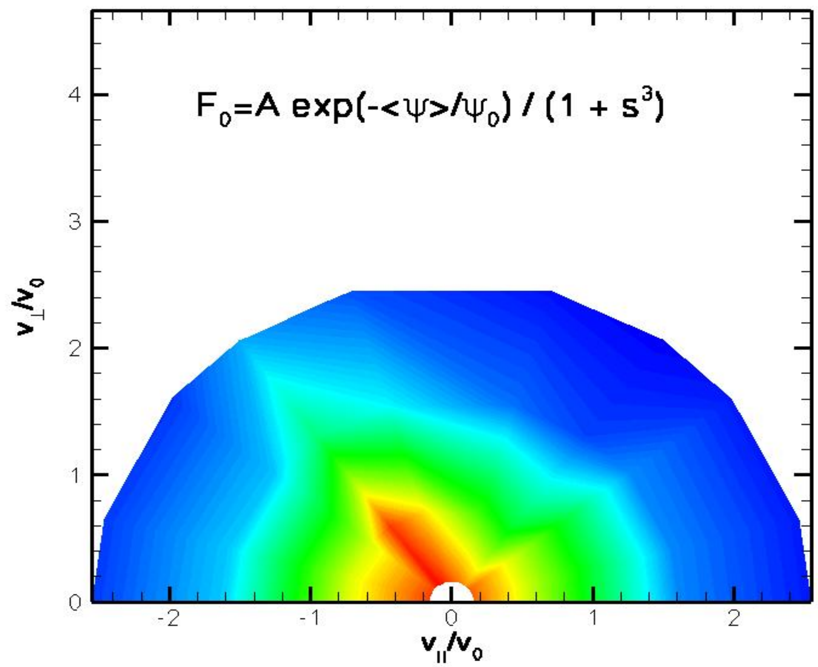
Lowest-order distribution functions, f_0, initialized in nimset.

General coding in field_comps and integrand routines permit easy testing of different f_0's.

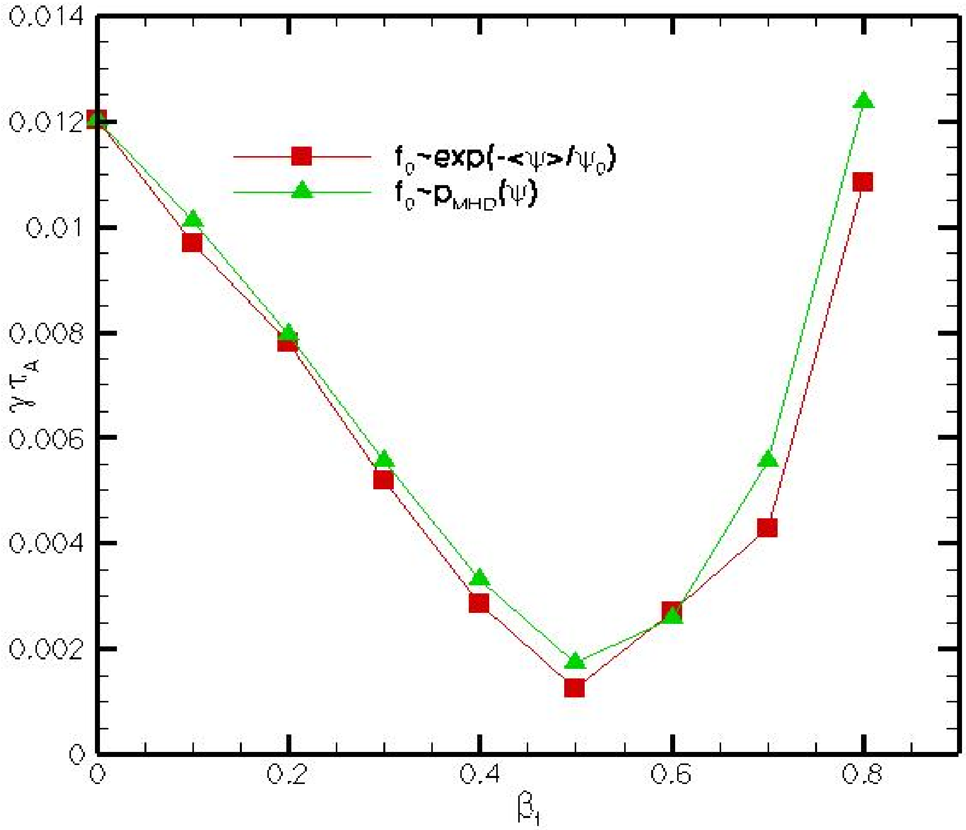
Anisotropic stress tensor for hot particles couples to NIMROD's momentum equation.

Examples of f_0 in DIII-D giant sawtooth simulations.





Linear growth rates insenstive to form of f_0.



Compare with predictions of NIMROD's δf-PIC algorithm; $f_0 \approx \exp\left(- < \psi > /\psi_0\right)$ hard-coded.

Add RF driven-tail to see if complete stabilization of ideal, internal kink is possible.

Add two-fluid effects and/or anisotropic stress closure for thermal ions.

Move forward with integral and continuum closures for PSI-Center.

Integral closures:

- closed forms that depend on NIMROD fluid moments which are readily available,
- valid for arbitrary collisionality and Z,
- do not include particle-trapping or time-dependent effects,
- require integration along magnetic fields lines; can reuse /closures/integral coding,
- are computationally intensive but memory light.

Continuum closures:

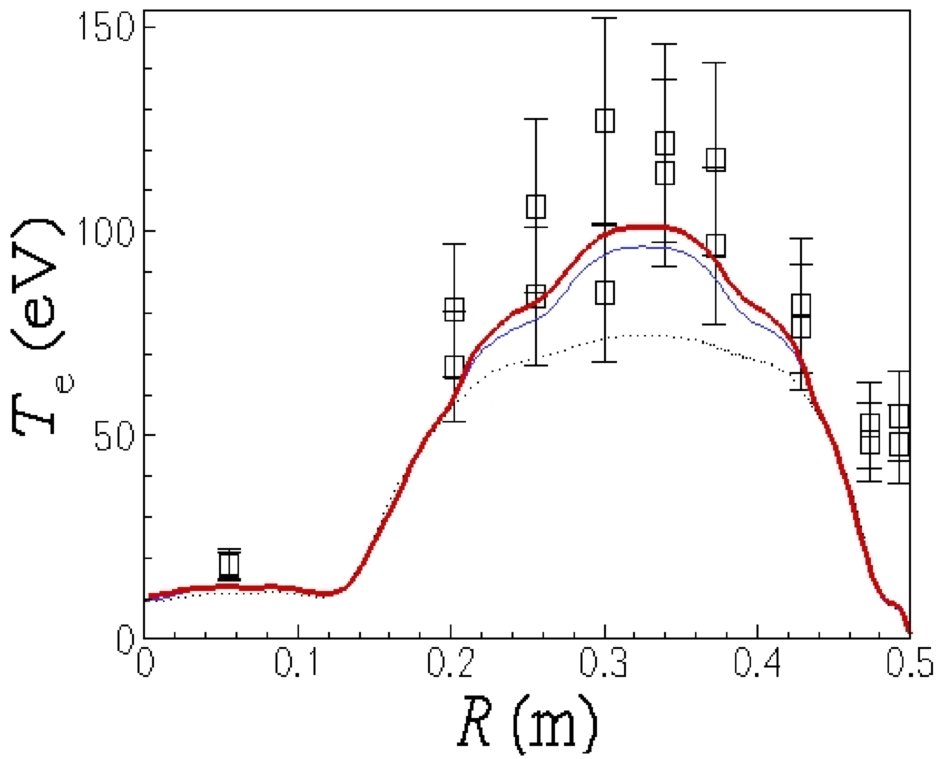
- computed as moments of solutions to 5D DKEs,
- valid for arbitrary collisionality and Z,
- include particle trapping and time-dependent effects,
- are fairly computationally efficient but memory heavy; great parallel scaling, though,
- although numerically efficient for linear computations, may be significantly more challenging nonlinearly.

One Year Plan - (from summer PSI-Center Meeting 2013)

NIMROD simulations of LTX experiment.

- Implement improved collision operator treatment. Done.
- Set up LTX equilibrium (piggyback on successful NIMROD CDX-U simulations). Work with Chris.
- Evolve electron DKE and couple $q_{||e}$ into T_e equation and $\pi_{||e}$ into Ohms Law. Compare predictions of continuum and integral electron closures with each other and the experiment.
- Evolve ion DKE and couple $\pi_{||i}$ into **V** equation. Could do it but is it necessary?

When are solutions to DKE's helpful?



- whenever Braginski with large coeffecients doesn't get parallel transport quite right (integral closures used in SSPX validation mildly helpful but computationally demoralizing),
- bootstrap current predictions for NTM's and H-mode edge physics (continuum),
- hot particle physics from neutral beams and/or fusion generated alphas (continuum).
- collisionless damping physics in reversed shear Alfven eigenmode calculations (continuum).

Need low-collisionality, well-diagnosed alternate experiments (tokamak-esque).